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On a mathematical model of simulating the hydraulic process of bars' formation at the Danube River mouths

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Abstract: The paper presents the description of the hydraulic process which takes place at the contact of the fluvial waters with the marine ones. The mathematical analysis of this hydraulic process is done and it results in two differential equations of the water flow. Both of them are related to the fluvial jets with variable discharge and constant discharge, respectively. The integration of the two differential equations was performed with finite differences at intervals of $dx = 100$ m, along the river jets, and two computing programs have been constructed in the Quick Basic 45 programming environment. Based on these programs, a mathematical model was constructed to simulate the fluvial sediment bars formation at the Sulina arm mouth where it meets the Black Sea waters. The results of simulation were compared to water depth measurements carried out within 1991-2009 on the central alignment of the bar. The model was tested using hydrological data from the 1978-2012 study interval, multiannual average monthly values of water discharges and coarse sediment transport, discharged by the Sulina arm mouth into the Black Sea. The model shows that the thickness of the bar alluvial layers deposited varies along the jets, extending off the sea up to a distance of about 1200 m. Based on the scenarios constructed in this model, the bar's dredging technology and the zones to be dredged can be established as the water has a depth to assure a good maritime navigation on the Sulina mouth bar.

Keywords: hydraulic process, hydraulic simulation, Danube's mouth, bar formation

INTRODUCTION

At the discharges of rivers in large aquatic objects such as oceans, seas and lakes, terrigenous alluvia transported by rivers are deposited forming natural sediment barriers called bars. The depositions occur as a result of the water energy decrease by dispersing the water masses into the aquatic space. These hydraulic processes are less known and varies from case to case by specific geological, hydrographical, and hydrological conditions of each mouth. The Danube mouths and the alluvial depositions processes fall into the category of contact of water masses with different densities. The Danube has relatively sweet water, with low mineralization, about 0.4‰ and the Black Sea up to 22‰. Theoretical basis for developing the mathematical model, presented in this paper, was taken from the PhD thesis " Bondar, C. 1972 - Contribution to the hydraulic study of output to the sea through the Danube mouths. In, *Hydrology studies*, vol. XXXII, Oceanography aspects, Institute of Meteorology and Hydrology, Bucharest, Romania.

Presentation of hydraulic process from the fluvial water contact with marine waters.

Before the presentation of hydraulic processes which occur at the contact of the Danube sweet water with the Black Sea salty water, there is necessary to present the hydraulic process that takes place along a stream of water drowned in a mass of water with the same density. The river water penetration into the marine environment takes place in the form of drowned water jets, along which friction is produced with the masses of lateral water and the seabed, for which the kinetic energy of the river jet is gradually spends. For this reason, once with the penetration into the marine environment of the river jet, there is gradually diminished the average speed of the water stream. By friction, the stream of water trains the masses of lateral water, which transmits part of the kinetic energy of the river jet. Thus, on the fluvial jet side, a moving water limit layer is formed, with the same direction of flow as the river jet and which contributes to the increase of the jet discharge.

The width of the jet increases gradually once with the penetration of the fluvial jet into the marine environment. Between the river jet and the marine water mass, a line of separation appears.

Figure 1 shows the flat kinematic scheme of the liquid jet drowned with free surface. Most of the experiences have shown that along the separator line vortexes are formed, which diffuse sideways, ensuring the mutual mixture of water masses. Due to the water masses friction with the sea bottom, there is neglected the influence of Coriolis acceleration in determining the equations of water flow. First, a mathematical analysis of the contact hydraulic process will be done in conditions where the masses of river and marine water have the same densities

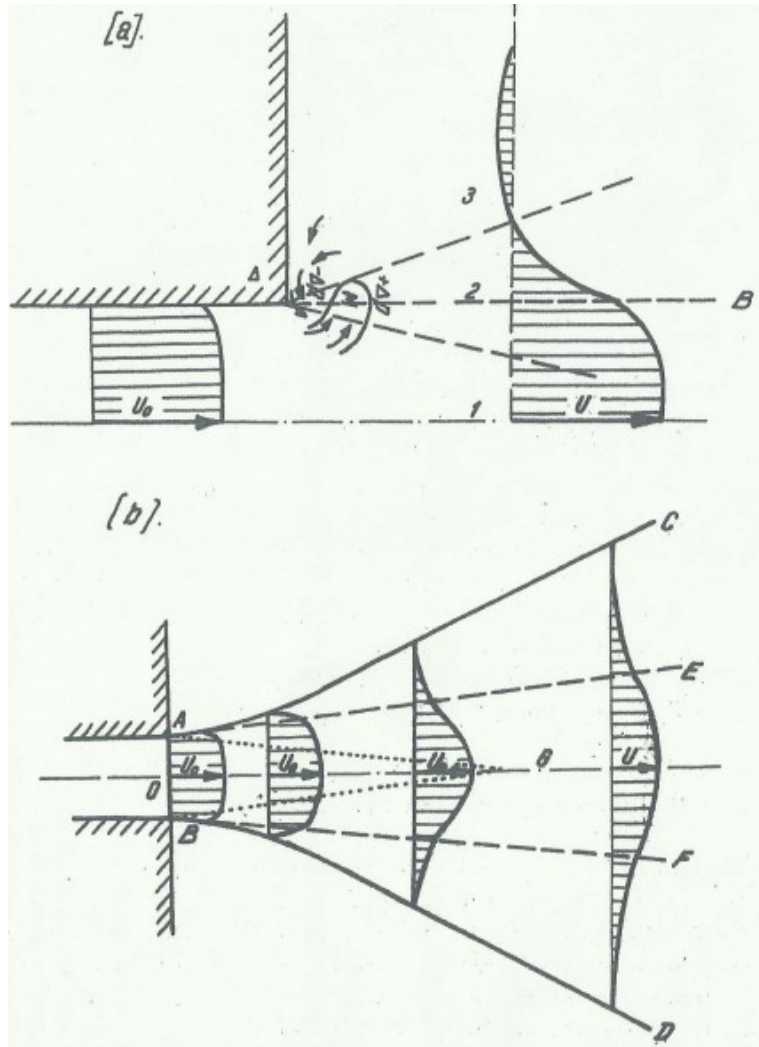


Figure 1. Flat kinematic scheme of the liquid jet drowned with free surface. (a) detail structural and (b) the whole movement, consisting of jet constant flow bounded by dashed lines AE and BF and with full jet variable flow, higher flow as constant, bounded by solid lines AC and BD.

Mathematical analysis of the hydraulic process

Based on diagrams from Figure 2, the hydrodynamic equilibrium equations of an elemental prism of water along the jet with variable discharge will be written.

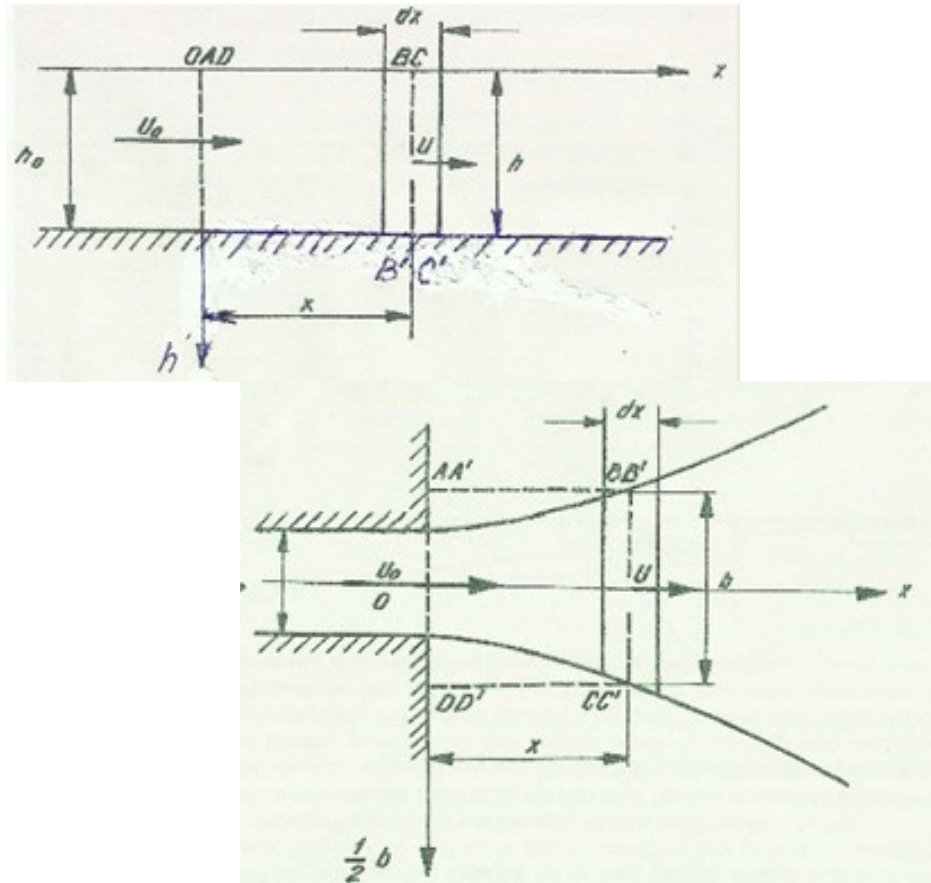


Figure 2. Diagram of liquid jet hydraulic model drowned in pool with liquid of the same density

Up, vertical profile along the jet.

Down, plan view of the jet.

Hydrodynamic processes and alluvial depositions, that produce bars at the Danube mouths, are analyzed and presented here for two conditions of water masses contact.

From the second law of mechanics, the dynamic equilibrium of forces in action is expressed by the equality (1).

$$dF = du / dt * dm \quad (1)$$

in which: dm - the mass of elementary prism of water from the liquid jet, u - the average speed of the prism along the jet, t - time, and dF - sum of external forces acting on the elementary prism of water from the jet.

The equality (1) components:

$$dm = \gamma / g * b * h * dx \quad (1.1)$$

in which; γ - specific weight of the water from the jet, g - gravitational acceleration, b - width of the jet, h - sea depth, and dx - elementary section of the jet.

$$dF = - DTL - DTF - dP \quad (1.2)$$

in which; DTL - lateral friction force of the water jet, DFF - Friction force on the sea bottom, and dP - gravity force.

$$DTL = \gamma / g * f_1 * u^2 / 2 * 2 * h * dx \quad (1.3)$$

$$DTf = \gamma / g * f_2 * u^2 / 2 * b * dx \quad (1.4)$$

$$dP = \gamma * b * h * dH / dx \quad (1.5)$$

in which: f_1 and f_2 - coefficients of friction with values of 0.172 and 0.00702 respectively, resulting from measurements in nature, h - sea depth, and dH / dx – water surface slope (has very low values and is neglected).

By using relations (1.1) - (1.5) in equation (1), there is obtained du (1.6):

$$du = - (f_1 * u^2 / b + f_2 * u^2 / 2 / h) * dx \quad (1.6)$$

The equation (1.6) expresses the balance of inertial forces of the jet and the side and bottom resistance. To solve it, there will be used the equation of the amount of movement that allows the deduction of the dependencies of variation, along the jet, of geometric and kinematic elements of the entire jet.

Thus, considering the figure 2, the control surface on the perimeter BCD, A', B', C', D' AA', DD', BB', CC', there is obtained an new equation from the projections equalization on OX, axes of pulses and pressures ADD on the sides ADD'A' and BCC 'B', in the form of (1.7).

$$\gamma / g * Q_0 * u_0 = \gamma / g * Q * U = \gamma / g * h * b * u^2 \quad (1.7)$$

Q_0 - the jet discharge in the mouth cross section.

The second relationship necessary in deduction dependencies of variations of the geometric elements and kinematics, for the jet part with constant discharge, is the continuity equation of the water discharge, as (1.8) form.

$$Q_0 = b_0 * h_0 * u_0 = b_1 * h_1 * u_1 \quad (1.8)$$

For the full jet, there is taken (b) from (1.7), which is inserted in the differential equation (1.6). Equation (1.9) results.

$$du = - [f_1 * h * u^3 / b_0 * h_0 * u_0^2 + f_2 / 2 * u / h] * dx \quad (1.9)$$

For the jet part with constant discharge along the jet, there is replaced in equation (1.9) the width (b) and it results in expression (1.10).

$$b_1 = b_0 * h_0 * u_0 / h / u_1 \quad (1.10)$$

obtaining differential equation 1.11)

$$du_1 = - U_1 * (f_1 * h * U_1 / B_0 / H_0 / u_0 + f_2 / 2 / h) * dx \quad (1.11)$$

Integration of water motion equations

On contact with the marine waters, fluvial waters penetrate them as drowned jet. Hydraulically, the phenomenon is described by the differential equations (1.9) for the jet with variable discharge and (1.11) for the constant discharge jet. The integration of the two differential equations was made with finite differences in the cases of non-loading and loading with alluvia of the river jet. For this purpose, two computing programs were developed in the programming environment Quick Basic 45. One of them is "MODBARJV.BAS", for variable discharge of the jet and the other one is "MODBARJC. BAS", for constant water flow.

The integration of differential equations was performed on intervals of distances $dx = 100$ m, from the River's mouth. Within each calculation program, the integration of differential equations solved the problem both in conditions of unloading with bucket and in terms of loading with bucket.

All results were tested with hydrological data - multiannual average monthly values from the 1978-2012 study interval of water flows and coarse bucket from the Sulina canal /arm mouth. Table 1 presents the monthly characteristic values, multiannual averages for the years 1978-2012 (maximum, average and minimum) of water flows at the mouth of Sulina canal.

Table 1. Monthly characteristic values (maximum, average and minimum) multiannual averages of discharges (mc /s) within 1978-2012, at the Sulina canal mouth.

Month	1	2	3	4	5	6	7	8	9	10	11	12	Annual average
Qmaxl	2505	1801	2369	3103	2956	2612	2262	1818	1661	1744	2000	1944	3103
Qmed	1279	1238	1441	1792	1682	1481	1233	997	889	921	991	1149	1258
Qminl	920	1121	1717	1769	1886	1937	1618	1086	967	1096	765	783	765

Accordingly, for the same study interval, there are presented in Table 2 monthly characteristic values (maximum, average and minimum) of coarse silt volumes spilled into the Black Sea through the Sulina canal /arm mouth.

Table 2. Multiannual average monthly values for the 1978-2012 study interval of the water flows, the volumes of river bottom gross bucket (Rgt), in suspension (Rgs) and total (Rg), spilled into the Black Sea by the Sulina canal /arm.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Qmed (mc/s)	1279	1238	1441	1792	1682	1481	1233	997	889	921	991	1149
Rgt (mc/month)	3215	3057	3837	5148	4739	3985	3038	2110	1675	1805	2086	2711
Rgs (mc/month)	10907	8228	23113	58673	46161	26530	7912	0	0	0	0	2974
Rg (mc/month)	14122	11285	26946	63820	50899	30514	10950	2110	1674	1804	2086	5685

Within 1978-2012, the annual average of the fluvial gross sediment transport at the Sulina mouth was about 221895 mc/year. Without details of the algorithm of integration with finite differences of the two differential equations, under conditions with and without sediment transport, the results obtained are presented.

From the field measurements, the results show a granulometric structure of the Danube mouths sediment transport consisting of coarse river sediment category (fine and medium-sized sands). This category was used to test the model. The input data in the two programs were: Sulina mouth (gsl), Sulina canal mouth width (b0 = 163 m), sea depth in the mouth area (h = 10 m), integration distance (dx = 100 m), and multiannual monthly average values of discharge and fluvial coarse sediment transport for the 1978-2012 study interval.

Variable water discharge

Without sediment transport

In this condition, each calculation program has determined from 100 to 100 meters, average speed values (u) and jet widths (b) up to 2000 m distance in the sea, along the river jet, resulting monthly a table of numerical values of speeds and widths as in the below example, for January (Table 3).

Table 3. Speed and width data calculated by the model: variable water flow, without sediment transport, and sea depth h = 10 m.

xi	uji	bji	u1ji	b1ji	xi	uji	bji	u1ji	b1ji
(m)	(m/s)	(m)	(m/s)	(m)	(m)	(m/s)	(m)	(m/s)	(m)
100	0.71	210	0.79	170	1100	0.38	729	0.53	251
200	0.63	262	0.76	177	1200	0.37	782	0.51	261
300	0.58	314	0.73	184	1300	0.35	836	0.49	271

400	0.54	365	0.70	191	1400	0.34	886	0.47	282
500	0.50	417	0.67	199	1500	0.33	937	0.46	293
600	0.47	469	0.65	207	1600	0.33	988	0.44	304
700	0.45	521	0.62	215	1700	0.32	1040	0.42	316
800	0.43	572	0.60	224	1800	0.31	1092	0.41	328
900	0.41	624	0.57	233	1900	0.30	1145	0.39	341
1000	0.39	676	0.55	242	2000	0.30	1198	0.38	354

With sediment transport

In this condition, using the "MODBARJV. BAS" computing program, there were determined, from 100 to 100 meters, average depth values (h) up to 2000 m off the sea, resulted from the coarse sediment transport of the Sulina arm (Table 4).

Table 4. Multiannual average monthly depths data within 1978-2012, from the calculation cross sections along the Sulina mouth bar, as calculated by the model, for the jet variable water flow condition.

Dis(m)	0	1	2	3	4	5	6	7	8	9	10	11	12
0		-9.57	-9.52	-9.75	-10.1	-10	-9.88	-9.51	-9.2	-9.05	-9.09	-9.19	-9.41
100	-10	-9.5	-9.07	-8.31	-6.79	-5.54	-4.7	-4.27	-4.25	-4.23	-4.21	-4.19	-4.02
200	-10	-9.92	-9.91	-9.57	-9.15	-8.81	-8.53	-8.53	-8.52	-8.51	-8.5	-8.5	-8.49
300	-10	-9.99	-9.99	-9.98	-9.63	-9.32	-9.3	-9.29	-9.28	-9.27	-9.26	-9.25	-9.24
400	-10	-9.99	-9.99	-9.98	-9.86	-9.86	-9.86	-9.85	-9.84	-9.83	-9.82	-9.81	-9.88
500	-10	-9.99	-9.99	-9.98	-9.98	-9.98	-9.97	-9.96	-9.95	-9.95	-9.94	-9.93	-9.92
600	-10	-9.99	-9.99	-9.98	-9.98	-9.97	-9.97	-9.96	-9.95	-9.95	-9.95	-9.94	-9.93
700	-10	-9.99	-9.99	-9.98	-9.98	-9.97	-9.97	-9.96	-9.95	-9.95	-9.95	-9.95	-9.94
800	-10	-9.99	-9.99	-9.98	-9.98	-9.97	-9.97	-9.96	-9.96	-9.96	-9.96	-9.96	-9.95
900	-10	-9.99	-9.99	-9.98	-9.98	-9.97	-9.97	-9.96	-9.96	-9.96	-9.96	-9.96	-9.95
1000	-10	-9.99	-9.99	-9.98	-9.98	-9.97	-9.97	-9.96	-9.96	-9.96	-9.96	-9.96	-9.96
1100	-10	-9.99	-9.99	-9.98	-9.98	-9.97	-9.97	-9.97	-9.97	-9.97	-9.97	-9.97	-9.97
1200	-10	-10	-10	-9.99	-9.99	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98
1300	-10	-10	-10	-9.99	-9.99	-9.99	-9.97	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98
1400	-10	-10	-10	-9.99	-9.99	-9.99	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98
1500	-10	-10	-10	-10	-10	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99
1600	-10	-10	-10	-10	-10	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99
1700	-10	-10	-10	-10	-10	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99
1800	-10	-10	-10	-10	-10	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99
1900	-10	-10	-10	-10	-10	-10	-9.99	-10	-10	-10	-10	-10	-10
2000	-10	-10	-10	-10	-10	-10	-9.99	-10	-10	-10	-10	-10	-10

The data from Table 4 are plotted in Figures 3 and 4 below.

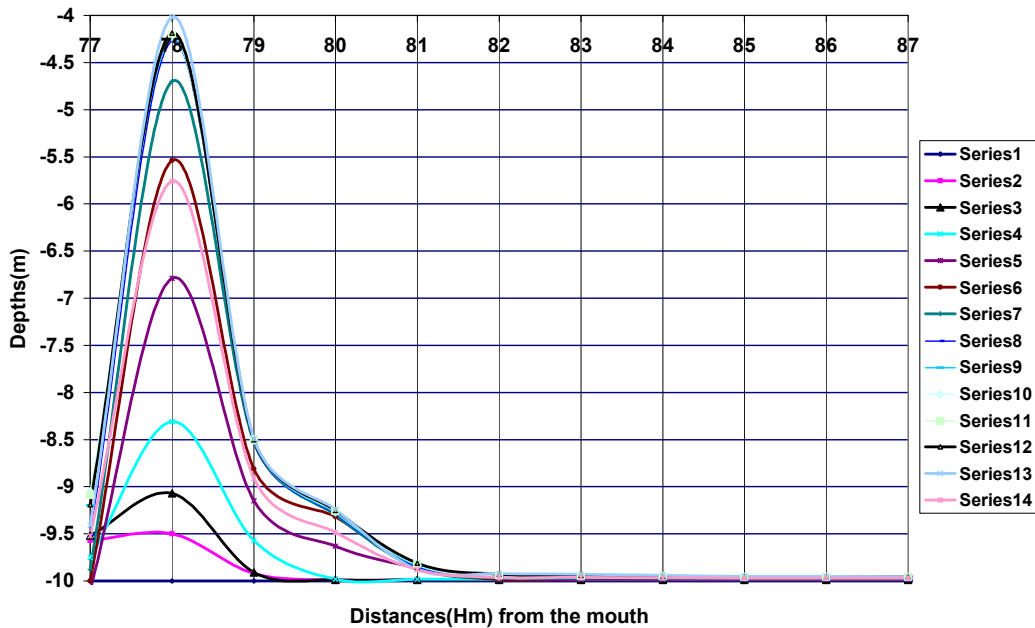


Figure 3. The monthly multiannual average depths variation graphs, for each 100 m cross-section, along the Sulina Canal mouth bar, in the jet variable discharge case, data for the study interval 1978-2012.

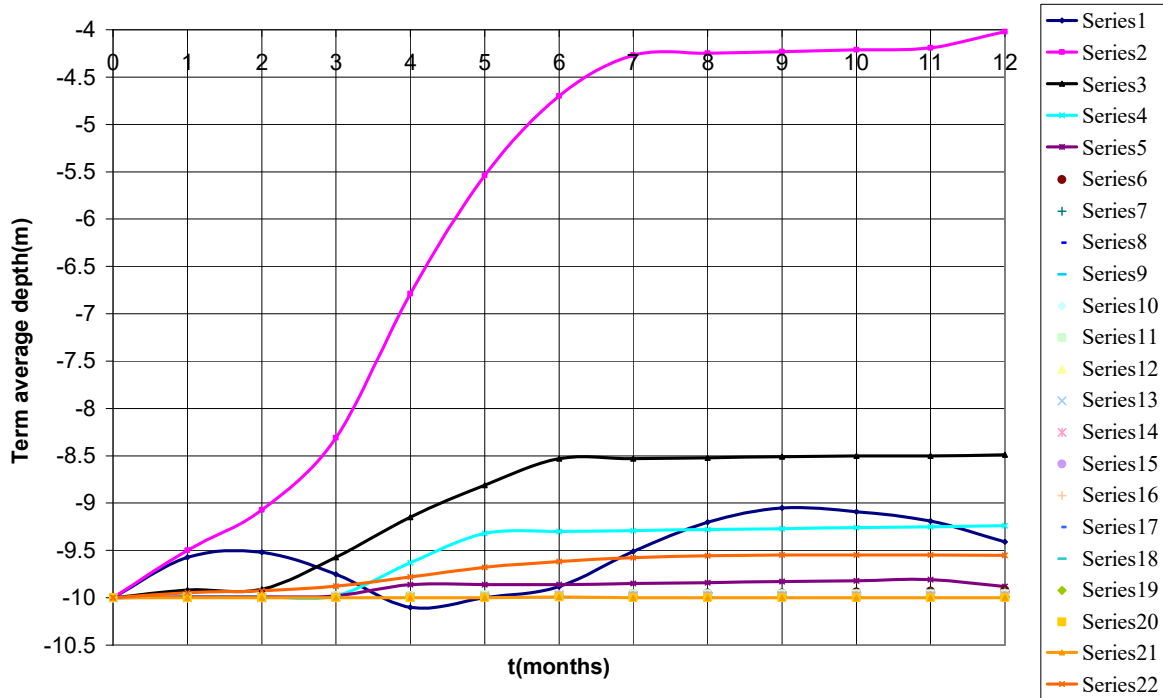


Figure 4. The annual variation of the multiannual monthly average depths, for each 100 m cross-section, along the Sulina Canal mouth bar, in the jet variable discharge case, data for the study interval 1978-2012.

Constant discharge

Using the "MODBARJC.BAS" computing program, there were calculated the multiannual average monthly depths from the calculation cross-sections along the Sulina mouth bar, modified in time and space of the Sulina canal coarse sediment transport, within 1978-2012, in constant water flow conditions (table 5).

Table 5. The data of the multiannual average monthly depths, calculated by the model in the calculation cross-sections, for each 100 m, along the Sulina Canal bar, in the case of constant water flow.

Dis(m)	0	1	2	3	4	5	6	7	8	9	10	11	12
0		-9.57	-9.52	-9.75	-10.1	-10	-9.88	-9.51	-9.2	-9.05	-9.09	-9.19	-9.41
100	-10	-9.5	-9.07	-8.31	-6.79	-5.54	-4.7	-4.27	-4.25	-4.23	-4.21	-4.19	-4.02
200	-10	-9.92	-9.91	-9.57	-9.15	-8.81	-8.53	-8.53	-8.52	-8.51	-8.5	-8.5	-8.49
300	-10	-9.99	-9.99	-9.98	-9.63	-9.32	-9.3	-9.29	-9.28	-9.27	-9.26	-9.25	-9.24
400	-10	-9.99	-9.99	-9.98	-9.86	-9.86	-9.86	-9.85	-9.84	-9.83	-9.82	-9.81	-9.88
500	-10	-9.99	-9.99	-9.98	-9.98	-9.98	-9.97	-9.96	-9.95	-9.95	-9.94	-9.93	-9.92
600	-10	-9.99	-9.99	-9.98	-9.98	-9.97	-9.97	-9.96	-9.95	-9.95	-9.95	-9.94	-9.93
700	-10	-9.99	-9.99	-9.98	-9.98	-9.97	-9.97	-9.96	-9.95	-9.95	-9.95	-9.95	-9.94
800	-10	-9.99	-9.99	-9.98	-9.98	-9.97	-9.97	-9.96	-9.96	-9.96	-9.96	-9.96	-9.95
900	-10	-9.99	-9.99	-9.98	-9.98	-9.97	-9.97	-9.96	-9.96	-9.96	-9.96	-9.96	-9.95
1000	-10	-9.99	-9.99	-9.98	-9.98	-9.97	-9.97	-9.96	-9.96	-9.96	-9.96	-9.96	-9.96
1100	-10	-9.99	-9.99	-9.98	-9.98	-9.98	-9.97	-9.97	-9.97	-9.97	-9.97	-9.97	-9.97
1200	-10	-10	-10	-9.99	-9.99	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98
1300	-10	-10	-10	-9.99	-9.99	-9.99	-9.97	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98
1400	-10	-10	-10	-9.99	-9.99	-9.99	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98	-9.98
1500	-10	-10	-10	-10	-10	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99
1600	-10	-10	-10	-10	-10	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99
1700	-10	-10	-10	-10	-10	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99
1800	-10	-10	-10	-10	-10	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99	-9.99
1900	-10	-10	-10	-10	-10	-10	-9.99	-10	-10	-10	-10	-10	-10
2000	-10	-10	-10	-10	-10	-10	-9.99	-10	-10	-10	-10	-10	-10

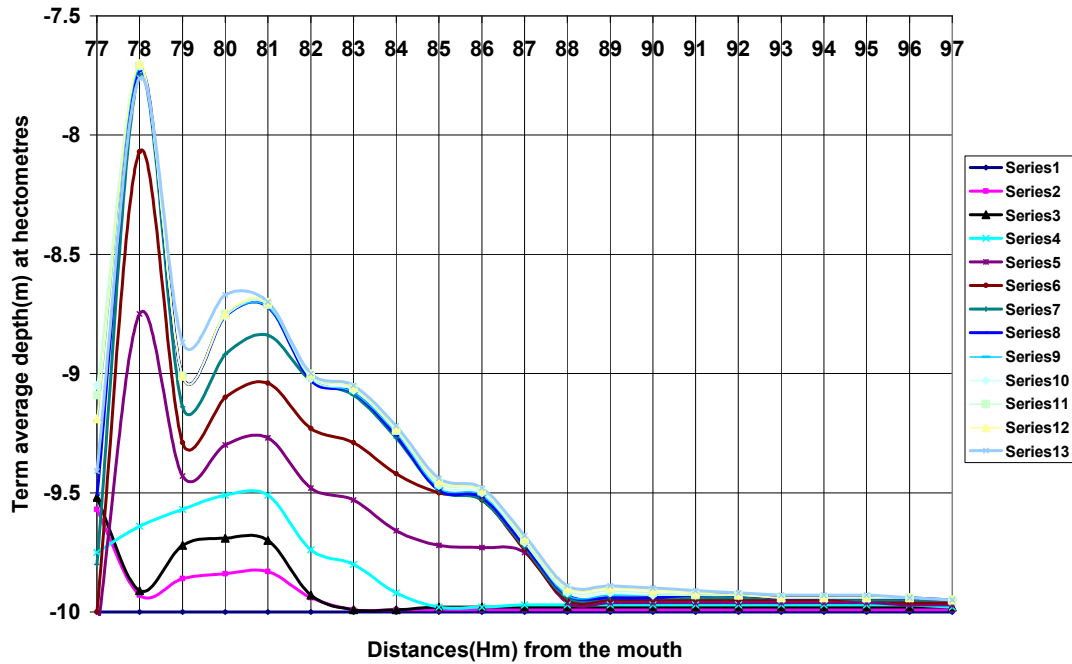


Figure 5. Multiannual monthly average depths variation graphs, within 1978-2012, in the calculation cross-sections, for each 100 m, along the Sulina mouth bar, for the jet constant discharge.

The graphs of Figure 5 show that there are gross alluvial depositions on the Sulina mouth bar without dredging intervention, they are deposited on the first 1100 meters.

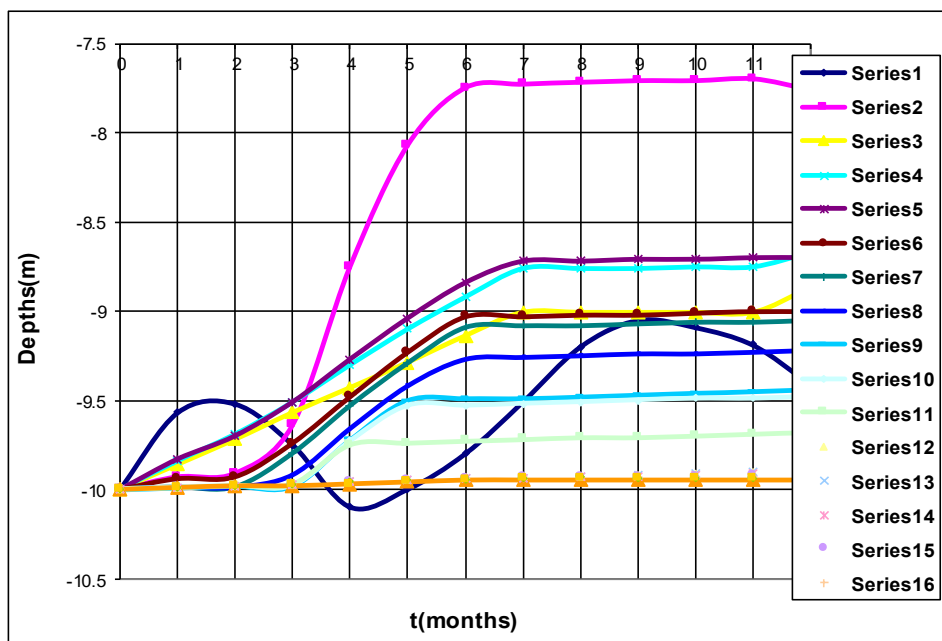


Figure 6. The annual variation graphs of the multianuale average monthly depths, within 1978-2012, in the calculation cross-sections, along the Sulina mouth bar, for the jet constant discharge conditions.

To compare the model results on the bar formation without dredging intervention, there is presented, in Figure 7, the situation of annual and multiannual average depths on the central alignment of the Sulina bar, in natural conditions of alluvial depositions with dredging intervention. Also, Figure 7 shows that by performing dredging works at the Sulina mouth bar, blockages of sediment deposits are cleared in the first few hundred of meters of the bar and the bar extends into the sea up to a distance of over 1000 m.

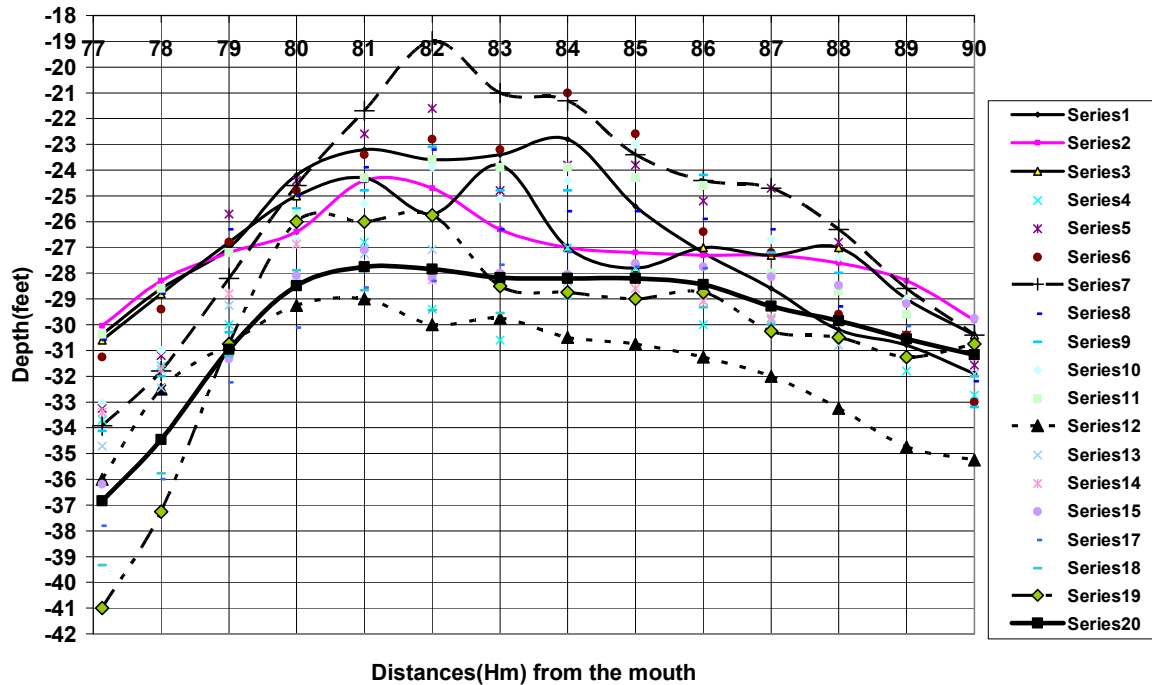


Figure 7. Annual average depths variation graphs, within 1991-2009 (1-19) and multiannual average (20), along the Sulina mouth bar.

Compared to the above data and information where the Black Sea salty water influences were ignored, in the below sections, they will be considered (saltwater, marine waves and currents) on the bars formation.

The Black Sea influence on the bars formation at the Danube mouths

There are three natural marine factors, salt water with density higher than the Danube water, waves and currents, which decisively influence the bars formation in the Danube mouths.

The influence of salt water

The marine salty water increases the energy of marine water compared to the river jet energy. From the application of the Bernoulli hydraulics law, in the Danube mouths cross-section, the following hydrodynamic equilibrium equation results (1.12):

$$V_f^2 / 2 / g + h / 2 = \gamma_m * h / 2 / \gamma_f \tag{1.12}$$

Where: V_f – the fluvial current average speed , g - gravitational acceleration, γ_f - specific weight of the water stream river bed, h – the river trough average depth, and γ_m - salty seawater specific weight.

Equation (1.12) can be written as expression (1.13)

$$V_f = [g * h * (\gamma_m - \gamma_f) / \gamma_f]^{0.5} \tag{1.13}$$

If the equation (1.13) refers to the mouth cross-section, it can be considered as a critical hydraulic

criterion of penetrating or not penetrating the sea saltwater in the river trough. When in the mouth cross-section the average speed of the river jet becomes smaller than the right side of the equation (1.13), there is the penetration of marine waters into the river trough and vice versa.

In order to obtain the critic average specific discharge, equality (1.13) is multiplied with h and it becomes (1.14).

$$q_{fcri} = V_f * h = [g * (\gamma_m - \gamma_f) / \gamma_f] * h^{1.5} \quad (1.14)$$

Using this criterion (1.14), there can be clarified the hydro-morphological processes of river coarse sediment deposition on the bar in the sea, in those conditions when the salt water surface is pushed to the sea by the river jet, beyond the river mouth cross-section. There will be presented these phenomena. The river jet emerging from its trough into the sea, will be subject to the sea water pressure at floating condition, beyond the critical cross-section as defined by the equation (1.14). Until the fluvial jet cross-section detachment from the sea bottom, the processes of deposition and bar formation will be subject to the hydraulic model laws, uninfluenced by the marine water environment, as presented in paragraphs 1-4.

CONCLUSIONS

The main results obtained by applying the mathematical model consist of the following:

1. Knowledge of the Danube River mouths bars formation under the coarse sediment transport into the Black Sea, in two water flow conditions, variable and constant discharge, for multi-annual average hydrological regime, as studied within 1978 – 2015.
2. If the jet variable water flow conditions, the bar formation occurs near the mouths, with a maximum blockage of deposited sediments in the first 200 meters, where the seabed rises with about 4.6 m in the cross-section located to 100 m from the mouth (Figure 3). The deposits expansion in the sea reaches the length of about 500 m. Bucket deposits increase from January to June.
3. In the jet constant water flow, the bar formation takes place on a larger stretch into the sea, reaching the length of about 1100 m (Figure 6). The maximum deposit blockage occurs in the same way, in the first 200 m, in which the sea bottom rises only by 2.3 m. The processes of alluvial depositions for the bars formation, are very close to the natural ones.
4. The model can be used to optimize the dredging technology on the Sulina mouth bar, by creating various scenarios of interaction of natural factors and techniques in action.
5. Also, the model allows to analyze the bars formation in various conditions of the Danube hydrological regime in order to deepen the knowledge of the alluvial deposits process in the Danube mouths.

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